Magnetic Circuits

Electric machines and transformers have electric circuits and magnetic circuits interlinked through the medium of magnetic flux. Electric current flow through the electric circuits, which are made up of windings. On the other hand, magnetic fluxes flow through the magnetic circuits, which are made up of iron cores. The interaction between the currents and the fluxes is the basic of the electromechanical energy conversion process that takes place in generators and motors. However, in transformers it is more feasible to think about the process in terms of an energy transfer. In transformers, the energy transfer is normally associated with change in voltage and current levels. Thus, magnetic circuits play an essential role.

1. Basics

The magnetic flux is produced due to the flow of a current in a wire (electric magnet). The direction of the produced magnetic flux is determined by "the right-hand rule" as shown in Fig. 1.

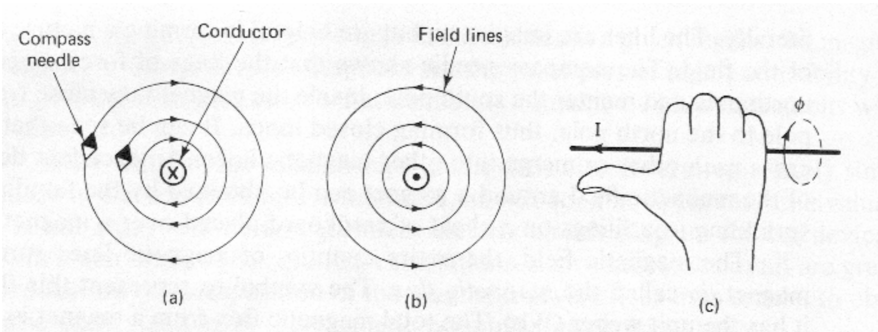


Fig. 1 The right hand rule for magnetic flux

The unit for the flux Φ is (weber) and the magnetic flux density B is given as:

$$B = \frac{\phi}{A} \text{ Wb/m}^2 \text{ (Tesla)}$$

The magneto-motive force mmf is the ability of a coil to produce magnetic flux. The mmf unit is Amp-turn and is given by: $mmf = NI\ (AT)$. The magnetic flux intensity H is the mmf per unit length along the path of the flux and is given by: $H = \frac{mmf}{l} \; (AT/m)$, where l is the mean or average path length of the magnetic flux in meters.

The relation between the mmf and the flux is governed by the system reluctance $\Re$, such that $mmf = \Re\phi$, where the reluctance is given by $\Re = \frac{l}{\mu A}$, where

l = The average length of the magnetic core (m)
A = The cross section area (m^2)
μ = The permeability of the material (AT/m^2)

The permeability of the material is given by

$$\mu = \mu_0 \mu_r$$

where: μ_0 is the permeability of air and μ_r is the relative permeability.

From the above relationships, we can conclude that:

$$B = \frac{\phi}{A} = \frac{mmf/\Re}{A} = \frac{(Hl)/(l/\mu A)}{A} = \mu H$$

The relation *B-H* is known as the magnetization characteristics of the material and is broken to three different regions: Linear, knee and saturation as shown in Fig. 2.

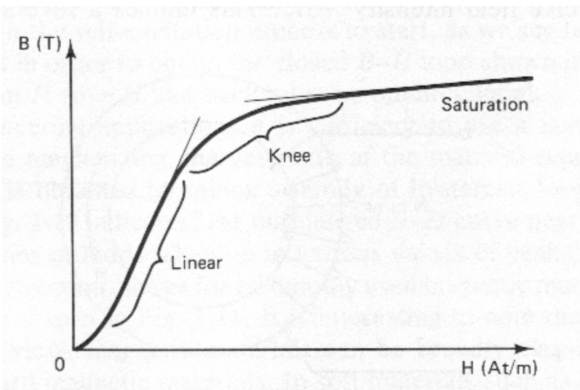


Fig. 2 Magnetization Curve

2. Analogy between magnetic and electric circuits

ELECTRIC CIRCUIT	*MAGNETIC CIRCUIT*
E = EMF	F = MMF
R = Resistance	$\Re$ = Reluctance
I = Current	ϕ = Flux
σ = Conductivity	μ = Permeability
$E = RI$	$mmf = \Re\phi$
$R = \frac{l}{\sigma A} = \frac{\rho l}{A}$	$\Re = \frac{l}{\mu A}$
$R_{series} = R_1 + R_2 + ... + R_N$	$\Re_{series} = \Re_1 + \Re_2 + ... + \Re_N$
$1/R_{parallel} = 1/R_1 + 1/R_2 + ... + 1/R_N$	$1/\Re_{parallel} = 1/\Re_1 + 1/\Re_2 + ... + 1/\Re_N$

3. Magnetic Circuit Analysis

In order to analyze any magnetic circuit, two steps are mandatory as illustrated by Fig. 3:

- ***Step #1:*** Find the electric equivalent circuit that represents the magnetic circuit.
- ***Step #2:*** Analyze the electric circuit to solve for the magnetic circuit quantities.

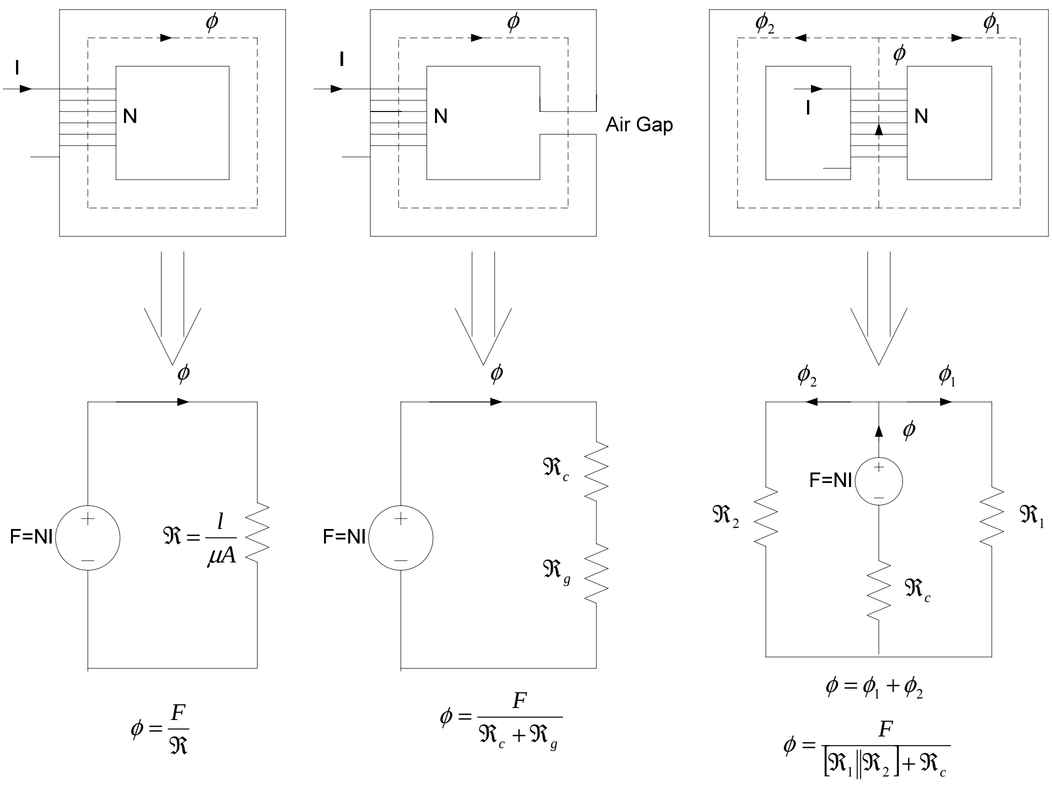


Fig. 3 Magnetic circuit analysis

Example

The magnetic circuit shown below has the following dimensions: A_c= 16 cm^2, l = 40cm, l_g = 0.5mm and N = 350 turns. The core is made of a material with the *B-H* relationship given below. For B = 1.0 T in the core, find:

a) The flux φ and the total flux linkage λ, where λ=N φ.

b) The required current to set this flux if there is no air gap.

c) The required current with the presence of an air gap.

B (Tesla)	H (A.T)
0.6	12.5
0.8	15.0
1.0	20.0
1.2	31.0
1.4	55.0

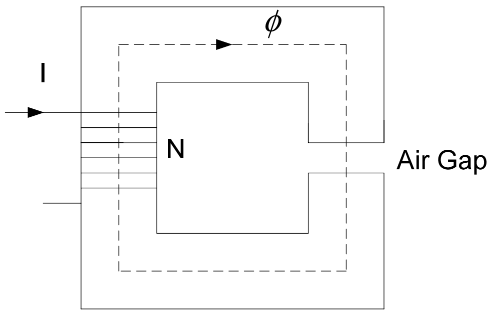


Fig. 4 Magnetic circuit for the example

Solution

a) $\phi = BA_c = 1.0 \times 16 \times 10^{-4} = 1.6\,mWb$

$\lambda = N\phi = 350 \times 1.6 \times 10^{-3} = 0.56\,Wb.t$

b) With no air-gap

$F = \Re_c \phi = NI$

$\therefore I = \dfrac{\Re_c \phi}{N}$

$\Re_c = \dfrac{l}{\mu_c A_c}$,

$$\mu_c = \frac{B}{H} = \frac{1.0}{20.0} = 0.05$$

$$\Re_c = \frac{40\times10^{-2}}{0.05\times16\times10^{-4}} = 5000\ At/wb$$

$$\therefore I = \frac{5000\times1.6\times10^{-3}}{350} = 22.86\ mA$$

c) With air-gap

$$F = NI = (\Re_c + \Re_g)\phi$$

$$\Re_c = \frac{l_c - l_g}{\mu_c A_c} \cong 5000,$$

$$\Re_g = \frac{l_g}{\mu_g A_g} = \frac{0.5\times10^{-3}}{(4\pi\times10^{-7})\times16\times10^{-4}} = 248{,}679.6$$

$$I = \frac{(\Re_c + \Re_g)\phi}{N} = 1.16\ A$$

In this example, it is clear that the current needed to set the same flux in case of magnetic circuits with air gap compared to those circuits without air-gap is much higher.

4. Fringing Effect

The fringing effect results from the presence of the air gap in the magnetic circuit. The main consequence of the fringing effect is to make the magnetic flux density of the air gap different from the flux density of the core due to the path of the flux.

$$\phi_c = \phi_g,$$

but $B_c \neq B_g$

Some times air gaps are introduced in the magnetic circuits to linearize the *B-H* curve. For a given current, the flux density will be smaller due to the air gap presence and saturation is not reached.

5. Leakage Flux

In order to set up a specific flux ϕ_c in the core, more MMF is needed due to the flux leakage phenomenon.

NOTE In most magnetic circuit analysis, both leakage flux and fringing effect are neglected.

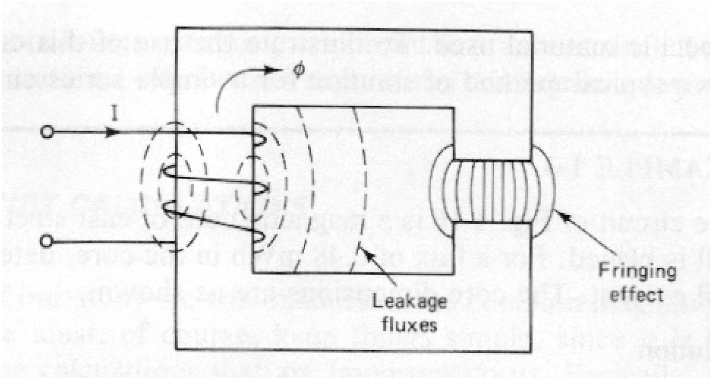


Fig. 5 Fringing effect and leakage flux

6. Core Losses (Iron Losses)

There are two types of losses that are present in the iron core of any transformer or rotating machine, namely hysteresis and eddy current losses.

6.1 Hysteresis Losses

Transformers and most rotating machines operate on alternating currents. In such devices, the flux in the iron changes continuously both in value and direction. The magnetic domains are therefore, oriented first in one direction, then the other, at a rate that depends upon the frequency. If we plot the flux density *B* as a function of the field intensity *H*, we obtain a closed curve called hysteresis loop as shown in Fig. 6.

In describing a hysteresis loop, the flux moves successively from $+B_m$, $+B_r$, *0*, $-B_m$, $-B_r$, *0*, and $+B_m$, corresponding respectively to points *a, b, c, d, e, f, and a* in the figure below. The magnetic material absorbs energy during each cycle and this energy is dissipated as heat. The amount of heat released per cycle is equal to the area of the hysteresis loop. To reduce the hysteresis losses, a magnetic material that have narrow hysteresis loop is normally selected.

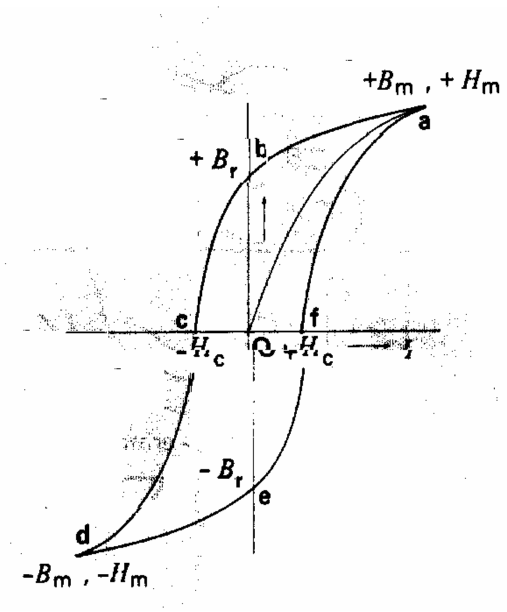


Fig. 6 Hysteresis loop

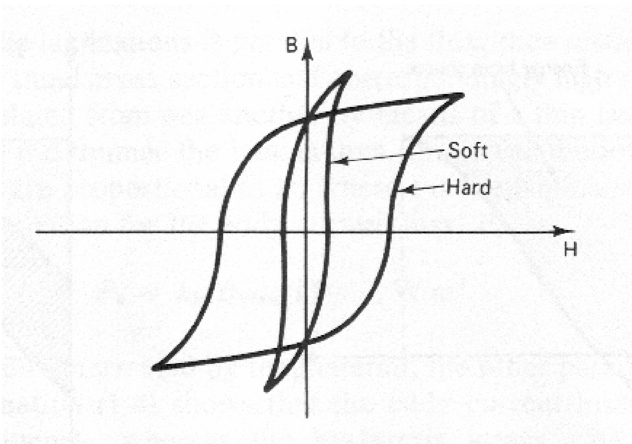


Fig. 7 Hysteresis loop for different materials

The hysteresis losses can be calculated using an empirical formula given as:

$$P_h = k_h \times V \times f \times B_{\max}^n$$

Where:

P_h	= The hysteresis losses (Watt)
k_h	= Hysteresis constant
V	= The material volume
f	= The excitation frequency
$B_{\max}$	= The maximum flux density
n	= Material constant (1.5-2.5)

6.2 Eddy Current Losses

The Eddy current problem arises when iron has to carry an ac flux. Fig. 8 shows a coil carrying an ac current that produces an ac flux in a solid iron core. Eddy currents (result due to the induced emf "Faraday's law") are set up as shown and they flow throughout the entire length of the core. The result of eddy currents is to heat up the core resulting in I^2R losses.

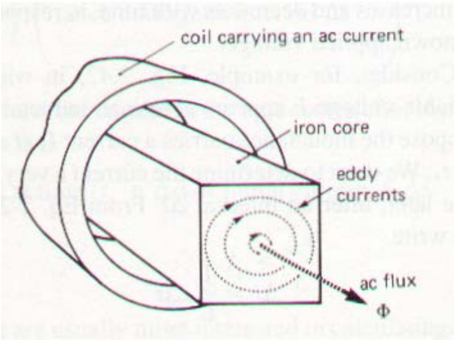


Fig. 8 Coil carrying an ac current

Eddy currents can be reduced by splitting the core in two along its length as shown in Fig. 9, taking care to insulate the two sections from each other. The voltage induced in each section is one half of what it was before, with the result of the Eddy current, and the corresponding losses are considerably reduced. If the core is further subdivided as shown in Fig. 10, the losses decrease progressively. In practice, the core is composed of lamination of 0.35-0.5mm in thickness.

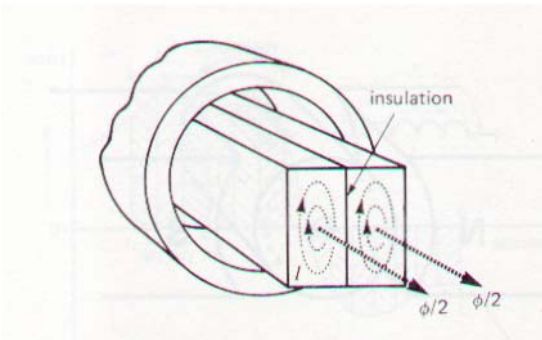


Fig. 9 Eddy currents in two section core

The Eddy current losses can be calculated using an empirical formula given as:

$$P_e = k_e \times V \times \left(f \times t \times B_{\max}\right)^2$$

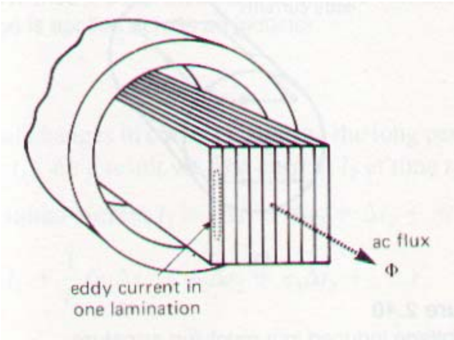


Fig. 10 Eddy current reduction when using laminated core

where:

P_e	= The Eddy current losses (Watt)
k_e	= Material dependent constant
V	= The material volume
f	= The excitation frequency
B_{max}	= The maximum flux density
t	= Lamination thickness

7. Inductances

The inductance (in Henry) is given by

$$L = \frac{\lambda}{I} = \frac{N\phi}{I}$$

Since: $F = NI = \phi\,\Re$,

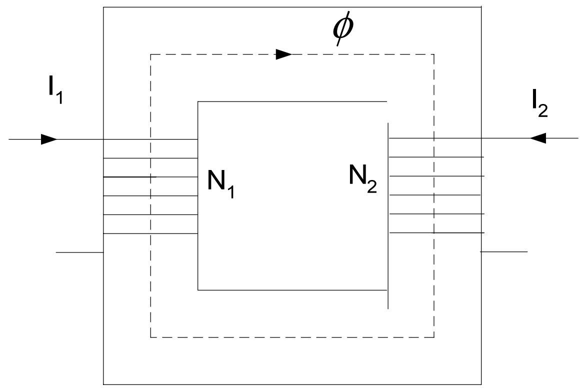
and $\Re = \dfrac{l}{\mu A}$

Therefore,

$$L = \frac{\lambda}{I} = \frac{N\phi}{I} = \frac{N^2 I \mu A}{I\,l} = \frac{N^2}{\Re}$$

$$L_{11} = \frac{\lambda_{11}}{I_1} = \text{self inductance}$$

$$L_{21} = \frac{\lambda_{21}}{I_1} = \text{mutual inductance}$$



where: λ_{11} is the total flux linking coil 1 due to I_1 when $I_2=0$, and λ_{21} is the total flux linking coil 2 due to I_1 when $I_2=0$

$$L_{11} = \frac{N_1^2}{\Re},$$

$$L_{22} = \frac{N_2^2}{\Re},$$

$$L_{21} = \frac{N_1 N_2}{\Re} = L_{12}$$

8. Faraday's Law

When a circuit is linked by a magnetic flux that is varying with time due to:

1. Relative motion of the magnetic field and the circuit
2. Time varying nature of the magnetic filed.

An emf will be induced in the circuit and is given by: $E = \frac{d\phi}{dt}$, which is known as Faraday's low for induction. The emf induced in the circuit can drive a current if this circuit is closed via a load. The direction of the induced current is such that the magnetic field generated by the current *OPPOSES* the magnetic field that caused the emf and the current. This is known as Lenz's law, which states that:

$$E = -\frac{d\phi}{dt}$$